Timing Errors in a Chain of Regenerative Repeaters, I

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The pulse displacements produced by timing errors in a chain of regenerative repeaters (using tuned-circuit timing filters) are represented by a linear transformation of the pulse displacements at the output of the first repeater. To facilitate the discussion of the general problem, the simpler case of periodic pulse trains is considered first. For this case it is shown that while the mean value tends to infinity, the central moments of the pulse displacements remain bounded as the number of repeaters approaches infinity. Further results are obtained which show that all the moments of the spacing jitter remain bounded for an indefinitely long string of repeaters. Finally, the misalignment in the jitter at any given repeater is represented by a simple expression which shows that the essential component in the misalignment is flat delay.

The general problem of random pulse trains, infinite in length, is discussed in Part II in this issue. The results obtained for the general case are quite different from those obtained for the periodic case. The variance is unbounded in this case except for pulse trains with certain special restrictions. The computational aspects for the evaluation of jitter accumulation will be discussed in a subsequent paper.

I. INTRODUCTION

In regenerative digital transmission systems, one of the important problems is that of maintaining the proper distance between the signal pulses. The problem becomes much more serious when the system contains a rather long chain of regenerative repeaters. Several aspects of a theoretical nature in connection with this problem have been discussed by Sunde,[1] Bennett,[2] Rowe[3] and Rice.[4]

We study here the pulse displacements produced by tuning errors in a chain of repeaters using tuned-circuit timing filters. For simplicity, we shall consider the system free of noise, distortion, etc.

An idealized version of the physical system is a chain of repeaters with the input supposed to be a train of unit impulses. Each repeater is a device containing a resonant circuit which is excited by the incoming train of pulses. The response of the resonant circuit to the incoming signal will ideally consist of a sum of sinusoids and will pass upwards through zero at an instant determined by the resonant frequency of the circuit. This instant will coincide with the instant of occurrence of the pulse, if it occurs at all, when the resonant frequency is identical with the pulse repetition frequency. The repeater does its "repeating" by sending out a unit impulse, at the instant the response of the tuned-circuit passes upwards through zero, provided the input signal has a pulse at or near the same instant. If there is no pulse in the input, no pulse is sent out.

Due to tuning error, the tuned circuit in a practical repeater would resonate at a frequency somewhat different from the pulse repetition frequency. Further, the impulse response of the circuit is more truly a damped sinusoid. These considerations show that the positions of the pulses sent out by a practical repeater are somewhat displaced from the true positions of the pulses in the original pulse train.

Actually the system consists of a chain of repeaters. We are thus led to a consideration of the statistical properties of the pulse displacements produced in a random pulse train by the combined effect of mistuning in each successive repeater. Of particular concern is the behavior of the pulse displacements as the number of repeaters gets larger and larger. It is to this question that we attend.

We begin our discussion by a mathematical statement of the problem. We show that the pulse displacements at the output of a chain of repeaters may be represented by a linear transformation, in a Banach space, of the pulse displacements at the output of the first repeater.

The linear operator (or, the linear transformation) becomes unbounded, in the limit, as the number of repeaters gets indefinitely large. From this follows the result that the average value* of the pulse displacements increases indefinitely as the number of repeaters approaches infinity.

The behavior of the variance, as well as the other central moments, of the pulse displacements is investigated by considering a suitable projection, when it exists, in the Banach space. When the domain of the above linear transformation is a linear manifold obtained by the desired projection, we find that the linear operator is bounded. Conse-

* All averages are taken over the values of the pulse displacements. No averages over the mistunings should be compared with the results obtained here.

quently, all the central moments of the pulse displacements are shown to remain bounded as the number of repeaters approaches infinity. When the above-mentioned projection does not exist, the central moments are shown to be unbounded.

Practical situations call for a determination of the bounds on the central moments when the number of repeaters is finite. In such cases, the input pulse trains may be assumed to be periodic pulse trains with the period much larger than the time constants of the timing filters. The problem reduces to a linear transformation in a finite dimensional vector space. The central moments are bounded and they can be precisely evaluated. A simple procedure to determine these bounds is developed.

The same analysis can be directly applied to an investigation of the so-called "spacing jitter," or variations in the spacings between virtual pulse positions. Similar results are obtained for both a finite and an infinite number of repeaters in the chain.

We shall also have occasion to remark upon the "misalignment noise" which is the jitter introduced, by the nth repeater, in an already jittered pulse train coming into the same repeater.

Finally, in a subsequent paper we shall discuss the computational aspects for the evaluation of jitter accumulation in a long string of repeaters.

Organization of the paper is as follows. We start with the statement of the problem in completely general terms and express it as a linear transformation. Next, to facilitate the discussion of the general problem, we consider the simpler case of a periodic pulse train. In Part II of the paper,* we consider the general case of a completely random pulse train.

II. STATEMENT OF PROBLEM

The input to the chain of repeaters is supposed to be a train of unit impulses which occur, if they occur at all, at the instants

$$\{\cdots, -2\tau, -\tau, 0, \tau, 2\tau, \cdots\}.$$

The occurrence or nonoccurrence of a pulse at time $t = -n\tau$ is determined by the value of the random variable $\mathcal{A}_n$. If $\mathcal{A}_n = 1$, which happens with a given probability, a pulse is present. If $\mathcal{A}_n = 0$, no pulse is present.

The resonant circuit in the repeater is excited by the incoming train

* Part II of the paper appears in this issue, p. 1781.

of pulses. The response of the circuit to a unit impulse at time $t = 0$ is assumed to be

$$e^{-\sigma t} \sin \omega_0 t, \qquad t > 0 \tag{1}$$

where $\omega_0 \tau$ is almost 2π but, due to tuning error, misses its desired value by

$$\epsilon = 2\pi - \omega_0 \tau = 2\pi(f_r - f_0)/f_r . \tag{2}$$

Here $f_r = 1/\tau$ is the pulse repetition frequency. The decrement σ is related to the Q of the circuit by $\sigma\tau = \pi/Q$.

The response of the resonant circuit to the incoming pulse train will consist of a sum of terms of the form (1) and will pass upwards through zero at an instant near $t = (-n\tau)$, say at $t = (-n\tau + t_n)$. The repeater sends out a unit impulse at the instant $(-n\tau + t_n)$ if the input signal has a pulse near $(-n\tau)$. If there is no pulse in the input, no pulse is sent out. The response of the resonant circuit still goes through zero, and we can say that there is a "virtual" pulse displacement of amount t_n seconds (or of $2\pi t_n/\tau$ radians).

For a chain of repeaters, we assume that all of the resonant circuits have the same Q but that their mistunings $\epsilon_1, \epsilon_2, \cdots$ are distributed independently and at random. Let $\xi_k{}^l$ be the displacement of the kth pulse (originally entering the first repeater at $t = -k\tau$) as it comes out of the lth repeater where $l = 1, 2, \cdots$. The displacement $\xi_k{}^l$ is measured in radians, where 2π radians corresponds to the pulse interval τ. The superscript l signifies the output of the lth repeater. The mistuning in the resonant circuit in the lth repeater is represented by ϵ_l. When we assume that Q is very large and the mistunings ϵ_l are much smaller than $\sigma\tau = \pi/Q$ radians, we are led to a set of equations which relate the pulse displacements out of the lth repeater to those out of the $(l - 1)$th repeater. These equations are

$$\xi_k{}^l = \frac{\sum_{n=0}^{\infty} \mathfrak{a}_{n+k}\beta^n(\xi_{n+k}{}^{l-1} + n\epsilon_l)}{\sum_{n=0}^{\infty} \mathfrak{a}_{n+k}\beta^n}, \tag{3}$$

$$(l = 1, 2, 3, \cdots; k = 0, 1, 2, \cdots),$$

where $\beta = \exp(-\sigma\tau) \approx 1 - (\pi/Q)$ is a number slightly less than unity. The initial conditions are that the pulses entering the first repeater have zero displacement, i.e.,

$$\xi_k{}^0 = 0, \qquad k = 0, 1, 2, \cdots . \tag{4}$$

These equations are given by Rowe[3] and also by Rice.[4] Here we have followed their terminology very closely.

The physical problem dealing with a chain of repeaters is now replaced by the mathematical problem of studying the behavior of the variables ξ_k^l defined by the above equations. The $\mathcal{A}_n$'s and ϵ_l's are either given explicitly or are random variables whose distributions are known.

III. LINEAR TRANSFORMATIONS

We note that the set of equations in (3) is a linear set, and we can express it as a linear transformation of the set of variables $\{\xi_k^{l-1}\}$ into the set $\{\xi_k^l\}$. We are, however, primarily interested in the behavior of $\{\xi_k^l\}$ when l is large and when no knowledge of $\{\xi_k^{l-1}\}$ is available. A more useful expression is obtained by rewriting (3) as

$$\xi_k^{\,l} = \frac{\sum_{n=0}^{\infty} \mathcal{A}_{n+k}\beta^n \xi_{n+k}^{\,l-1}}{\sum_{n=0}^{\infty} \mathcal{A}_{n+k}\beta^n} + \frac{\epsilon_l}{\epsilon_1}\,\xi_k^{\,1}, \tag{5}$$

where

$$\xi_k^{\,1} = \frac{\sum_{n=0}^{\infty} \mathcal{A}_{n+k}\beta^n n\epsilon_1}{\sum_{n=0}^{\infty} \mathcal{A}_{n+k}\beta^n}. \tag{6}$$

In our formulation, zero mistuning does not introduce any jitter in a jitter-free pulse train. We will therefore understand the chain to start with a repeater having non-zero mistuning.

Equation (5) can be used to express $\{\xi_k^l\}$ as a linear transformation of $\{\xi_k^1\}$. To do this, define a matrix (infinite)

$$T = \begin{bmatrix} \frac{\mathcal{A}_0}{s_0} & \frac{\mathcal{A}_1\beta}{s_0} & \frac{\mathcal{A}_2\beta^2}{s_0} & \cdots & \\ 0 & \frac{\mathcal{A}_1}{s_1} & \frac{\mathcal{A}_2\beta}{s_1} & \frac{\mathcal{A}_3\beta^2}{s_1} & \cdots \\ 0 & 0 & \frac{\mathcal{A}_2}{s_2} & \frac{\mathcal{A}_3\beta}{s_2} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \end{bmatrix}, \tag{7}$$

where

$$s_i = \sum_{n=0}^{\infty} \alpha_{n+i}\beta^n; \tag{8}$$

and define a vector

$$X_l = [\xi_0^l, \xi_1^l, \xi_2^l, \cdots]. \tag{9}$$

Then (5) becomes

$$X_l = TX_{l-1} + \frac{\epsilon_l}{\epsilon_1} X_1; \qquad (X_0 = 0), \quad (l = 1, 2, 3, \cdots). \tag{10}$$

From (10) it follows that

$$X_l = \left[\frac{1}{\epsilon_1} \sum_{\nu=0}^{l-1} \epsilon_{l-\nu} T^\nu\right] X_1, \quad (T^0 = I). \tag{11}$$

One can, if need be, discuss the behavior of (11) in the above form. However, the ϵ's are usually of the same order of magnitude, and the equation is considerably simplified by assuming tbat the ϵ's are identical.* Then

$$X_{l+1} = \left[\sum_{\nu=0}^{l} T^\nu\right] X_1. \tag{12}$$

We are interested in the problem when l becomes indefinitely large, or, dropping superfluous subscripts,

$$Y = \lim_{l\to\infty} \left[\sum_{\nu=0}^{l} T^\nu\right] X. \tag{13}$$

Here X and Y represent the pulse deviations out of the first repeater and out of the $(l + 1)$th repeater, respectively.

The original problem is now represented as a linear transformation of X into Y. The linear transformation, when it exists, is a function of another linear transformation T. The domain, as well as the range, of the transformation T is a Banach space, as will be shown in Part II. Here, we pursue the simpler case of a periodic pattern.

Whether the variance is bounded or not is not a particularly important question for the periodic case. Such a question can be answered by a very simple argument. However, we give here instead a complete analysis of the periodic case. Our purpose in doing so is twofold. First, the analysis shows how certain basic properties of the operator T influence the questions of boundedness of the jitter; it also gives a simple

* We shall discuss elsewhere the difference, if any, in the results when we do not make this assumption.

computational procedure for evaluating the accumulated jitter. Second, the analysis serves as a simple introduction to the more complex argument pursued in Part II.

IV. PERIODIC PULSE TRAINS

We assume here that the $\mathcal{a}_n$'s form a pattern which repeats itself with a period m. The pattern is otherwise arbitrary. In such cases, the pulse displacements are also periodic with the same period m. Then,

$$\begin{aligned} \mathcal{a}_{n+m} &= \mathcal{a}_n \\ \xi_{k+m}{}^{l} &= \xi_k{}^{l} \end{aligned} \tag{14}$$

for all values of indices n and k.

The domain of the operator T is thus an m-dimensional space. Since $\mathcal{a}_{n+m} = \mathcal{a}_n$, the range of T is also of dimension m. The problem reduces to the study of a linear transformation in a finite dimensional space. The operator T is now represented by a finite matrix A.

$$A = \begin{bmatrix} \left(\dfrac{\mathcal{a}_0 + \mathcal{a}_m\beta^m + \mathcal{a}_{2m}\beta^{2m} + \cdots}{s_0}\right) & \left(\dfrac{\mathcal{a}_1\beta + \mathcal{a}_{m+1}\beta^{m+1} + \cdots}{s_0}\right) & \cdots \\ \left(\dfrac{\mathcal{a}_m\beta^{m-1} + \mathcal{a}_{2m}\beta^{2m-1} + \cdots}{s_1}\right) & \left(\dfrac{\mathcal{a}_1 + \mathcal{a}_{m+1}\beta^m + \cdots}{s_1}\right) & \cdots \\ \cdots & \cdots & \cdots \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{\mathcal{a}_0}{s_0'} & \dfrac{\mathcal{a}_1\beta}{s_0'} & \cdots & \dfrac{\mathcal{a}_{m-1}\beta^{m-1}}{s_0'} \\ \dfrac{\mathcal{a}_0\beta^{m-1}}{s_1'} & \dfrac{\mathcal{a}_1}{s_1'} & \cdots & \dfrac{\mathcal{a}_{m-1}\beta^{m-2}}{s_1'} \\ \vdots & & & \vdots \\ \dfrac{\mathcal{a}_0\beta}{s_{m-1}'} & & & \dfrac{\mathcal{a}_{m-1}}{s_{m-1}'} \end{bmatrix}, \tag{15}$$

where

$$s_k' = (1 - \beta^m)s_k . \tag{16}$$

For the periodic case, (13) becomes

$$Y = \lim_{l\to\infty} \left[\sum_{\nu=0}^{l} A^\nu\right] X, \tag{17}$$

where we continue to use the same symbols X and Y to represent the finite dimensional vectors.

In order to investigate the limit of (17), one must determine the behavior of the infinite series and its convergence properties. Moreover, if the limit does not exist, the question to be answered is whether or not the variance of Y has any limit. Other central moments may also be of interest.

In what follows, we show that the limit of (17) does not exist. This implies that the mean of Y is infinite. However, we shall show that the central moments always exist for any arbitrary m. We assume throughout this paper that averages over the sample values* are statistically identical to the averages over the ensembles.

A discussion of the properties of the linear transformation defined by (17) involves the study of a function of the matrix A. In order to discuss such a function, one must have a knowledge of the spectrum of the matrix. We study the spectrum of A in the next section.

V. SPECTRUM OF A

In this section, we prove the following theorem.

Theorem: The spectrum of A consists of two parts:

1. The maximum eigenvalue is located at $\lambda = 1$, and it is simple;

2. All other eigenvalues are such that their modulus is less than unity. i.e., $|\lambda_i| < 1$.

Proof: Observe that A is a stochastic matrix since the sum of each row is equal to one and all the elements of the matrix are nonnegative. Thus, $\lambda = 1$ is indeed an eigenvalue with eigenvector $\{1, 1, \cdots, 1\}$. It also follows that the entire spectrum of A is contained in the unit disk $|\lambda| \leqq 1$. This can be observed in a simple manner by considering powers of matrix A and noting that the trace of A^n does not exceed m, the order of the matrix A. If there were any eigenvalue for $|\lambda| > 1$, one could find a large enough n such that the trace of A^n would exceed m. (We do not worry about cancellation because we can always choose the proper n to prevent this.) Hence, there are no eigenvalues outside the unit disk.

Next, we wish to show that there are no other eigenvalues ($\lambda \neq 1$) with modulus equal to one. We obtain a matrix equivalent to A by means of elementary transformations of interchanging rows as well as the corresponding columns. The eigenvalues of the matrix are invariant

* The values of the pulse displacements are referred to as the sample values, and the ensemble is the set of admissible sequences of pulse displacements. For justification of the above assumption in the general case, see Bennett, op. cit.

under such operations. We obtain a matrix of the form

$$B = \begin{bmatrix} A' & 0 \\ C & D \end{bmatrix}, \tag{18}$$

where A' is a square matrix all of whose elements are positive and D is a square null matrix. Only the eigenvalues of A' need be considered. To A' we apply Perron's theorem which, for a stochastic matrix with all elements positive, states that: the extremum eigenvalue is located at $\lambda = 1$; it is simple; and its modulus exceeds the moduli of all other eigenvalues. Q.E.D.

VI. MEAN, VARIANCE, ETC.

The solution to (17) can now be expressed in terms of the basis vectors of A in the form

$$Y = \lim_{l \to \infty} \sum_{\nu=0}^{l} \sum_{\mu=1}^{m} \lambda_\mu{}^\nu \alpha_\mu X^{(\mu)}, \tag{19}$$

where, $X^{(\mu)}$ is the eigenvector of A corresponding to the eigenvalue λ_μ of A. The coefficients α_μ are the expansion coefficients in

$$X = \sum_\mu \alpha_\mu X^{(\mu)}. \tag{20}$$

We have assumed, for the present, that A is of simple structure. There are no significant changes in the development when such an assumption is not made. We shall discuss this matter a little later.

In the previous section it has been proved that the extremum eigenvalue, say λ_1, is simple and is located at $\lambda_1 = 1$. The rest of the eigenvalues are strictly inside the unit circle. The mean value of Y is seen to approach infinity by considering only those terms that involve $\lambda_1 = 1$,

$$\bar{Y} = \alpha_1 X^{(1)} \sum_{\nu=0}^{\infty} \lambda_1{}^\nu + \sum_{\mu=2}^{m} \frac{\alpha_\mu}{1 - \lambda_\mu} \overline{X^{(\mu)}}, \tag{21}$$

where, $X^{(1)} = \{1, 1, \cdots, 1\}$.

The first term in (21) is a divergent series and $\bar{Y}$ approaches infinity as the number of repeaters increases indefinitely.* The behavior of the central moments is investigated by considering

$$[Y - \bar{Y}] = \sum_{\mu=2}^{m} \left(\frac{\alpha_\mu}{1 - \lambda_\mu}\right) [X^{(\mu)} - \overline{X^{(\mu)}}]. \tag{22}$$

* The statement is valid, in general, provided $\alpha_1 \not\equiv 0$. We need only show that there exists at least one X such that $\alpha_1 \neq 0$. Consider a pulse train with all pulses present; then $X = \alpha_1 X^{(1)}$ with $\alpha_1 \neq 0$.

The value of $\overline{[Y - \bar{Y}]}$ is finite since $|\lambda_\mu| < 1$. All powers of $[Y - \bar{Y}]$ are finite and $\overline{[Y - \bar{Y}]^k}$ will also be finite. We thus see that all the central moments, including the variance, are finite.

Now we consider the case when the structure of A is not simple. The only difference in this case concerns the vectors corresponding to eigenvalues other than λ_1. Let us, for simplicity, consider the basis vectors that correspond to an eigenvalue λ_μ of multiplicity two. Similar developments can be carried out when the multiplicity is greater than two. The normal form of A would have a Jordan block

$$\begin{bmatrix} \lambda_\mu & 1 \\ 0 & \lambda_\mu \end{bmatrix} \tag{23}$$

It is well known that there exist two linearly independent vectors $X_1^{(\mu)}$ and $X_2^{(\mu)}$ such that

$$\begin{aligned} AX_1^{(\mu)} &= \lambda_\mu X_1^{(\mu)} \\ AX_2^{(\mu)} &= \lambda_\mu X_2^{(\mu)} + X_1^{(\mu)}. \end{aligned} \tag{24}$$

The vector $X_1^{(\mu)}$ is an eigenvector of A and is transformed in the same manner as the vectors $X^{(\mu)}$ are, and it yields for Y a term of the form

$$\left(\frac{\alpha_\mu}{1 - \lambda_\mu}\right) X_1^{(\mu)}. \tag{25}$$

On the other hand, when A^ν operates on $X_2^{(\mu)}$ it yields

$$A^\nu X_2^{(\mu)} = \lambda_\mu^{\ \nu} X_2^{(\mu)} + \nu\lambda_\mu^{\ \nu-1} X_1^{(\mu)}. \tag{26}$$

Thus $X_2^{(\mu)}$ contributes to Y a term of the form

$$\begin{aligned} \alpha_\mu' \sum_{\nu=0} A^\nu X_2^{(\mu)} &= \alpha_\mu' \left[\sum_{\nu=0}^{\infty} \lambda_\mu^{\ \nu} X_2^{(\mu)} + \sum_{\nu=0}^{\infty} \nu\lambda_\mu^{\ \nu-1} X_1^{(\mu)} \right], \\ &= \alpha_\mu' \left[\left(\frac{1}{1 - \lambda_\mu}\right) X_2^{(\mu)} + \left(\frac{1}{1 - \lambda_\mu}\right)^2 X_1^{(\mu)} \right], \end{aligned} \tag{27}$$

since $|\lambda_\mu| < 1$.

The terms due to the basis vectors of A corresponding to λ_μ are shown to be bounded, and our results on the boundedness of the central moments remain valid regardless of the structure of A.

VII. SPACING AND MISALIGNMENT

Sometimes a more useful measure of jitter is the spacing jitter, which is defined as the deviations in the spacing between adjacent pulses or

pulse positions. This is obtained by taking the difference of the adjacent pulse position deviations. To do this operationally, let us define an operator S which shifts the elements in the vector Y such that the kth element appears as the $(k - 1)$th element and the first element appears as the mth element.

$$S = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & & & & & \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{28}$$

The spacing jitter Y_s can then be represented in terms of the timing jitter Y by

$$Y_s = [I - S]Y. \tag{29}$$

By using (19) and (29), we have

$$Y_s = \lim_{l \to \infty} \sum_{\nu=0}^{l} \sum_{\mu=1}^{m} \lambda_\mu^{\nu} \alpha_\mu [I - S] X^{(\mu)}. \tag{30}$$

The operator $[I - S]$ annihilates $X^{(1)}$ and we obtain

$$Y_s = \sum_{\mu=2}^{m} \left(\frac{\alpha_\mu}{1 - \lambda_\mu} \right) [I - S] X^{(\mu)}. \tag{31}$$

The spacing jitter is finite for all sample values, and so the mean and all other moments of this jitter are finite.*

Next, we briefly consider the misalignment which is defined as the difference between the timing errors at the output and at the input of a given repeater. The representation of the misalignment in the $(l + 1)$th repeater is given, in the periodic case, by modifying (12) to a finite dimensional one and obtaining

$$[X_{l+1} - X_l] = A^l X_1 , \tag{32}$$

where, X_k represents the jitter at the output of the kth repeater.

Equation (32) implies that the misalignment essentially amounts to a flat delay as l gets larger. Indeed, there is virtually no difference in the misalignment for different repeaters when the values of l are reasonably large.

* For periodic pulse patterns, this is intuitively obvious.

VIII. CONCLUSION

The general problem of timing errors in a string of repeaters has been expressed in terms of certain linear operators and functions of these operators. The simpler case of periodic pulse patterns is then studied in detail. We have shown, for the periodic case, that the mean value of jitter accumulation in a string of repeaters increases indefinitely but that the central moments of the jitter remain bounded. In fact, the divergence of the mean value for the infinitely long string stems from the accumulation of the flat delay occurring in each repeater. Once this flat delay is eliminated, the remaining part of the jitter is bounded. Consequently, all the central moments are bounded. All the moments of the spacing jitter are bounded for identical reasons. The misalignment behavior is also explained by the dominance and the invariance of the flat delay.

The question of evaluating the jitter accumulation will be discussed in a subsequent paper. We will show there that the spectrum of the operator A can be determined fairly simply even for very large periodicity. No polynomials of high degrees need be solved to determine the eigenvalues. We shall also discuss the computation errors involved in periodic approximation versus those involved in truncation of the infinite pulse train.

The general case of random pulse trains with no periodic structure will be examined in Part II. We shall have occasion to thoroughly examine the operator T. Since we shall be concerned with infinite dimensional space, the spectral properties of T are not so easy to determine. We shall compare the spectral properties of T with those of A in order to delineate the difference between the two cases.

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